



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

WHAT MATHEMATICAL KNOWLEDGE AND ABILITY MAY REASONABLY BE EXPECTED OF THE STUDENT ENTERING COLLEGE?

BY JAMES N. HART.

If we contemplate the numerous committee reports of this and other societies upon the teaching of arithmetic, algebra, and geometry, together with the excellent syllabi published by them, it would seem that our question has been so fully and definitely answered, that further discussion is uncalled for, if not almost impertinent. If, on the other hand, we turn to the series of papers presented before the Association of Teachers of Mathematics for the Middle States and Maryland last year and printed in the *MATHEMATICS TEACHER* for March and June of the present year upon the question, "What Mathematical Subjects Should Be Included in the Curriculum of the Secondary School," it is very evident that the doctors still disagree. At one extreme we find a speaker, a college graduate, but not, according to his own confession, a mathematician, arguing that so few students are "mathematically minded" that it would be more profitable to limit high school mathematics to arithmetic and allow the few who reach college to elect algebra and geometry—if they have any remaining curiosity regarding mathematics. At the other extreme, a high school principal presents an "outline of mathematical work that should be required of every student in a general high school course," including not only the traditional work in algebra and plane geometry, but solid geometry, trigonometry and applications of algebra to mechanics, science, economics, statistics, shop mathematics and the slide rule. And again, when we consider the propositions of the committee on articulation of the N. E. A. and those of the commissioner of education of the state of Massachusetts we realize that the question is not finally settled, and that we mathematicians will not be allowed to settle it by ourselves.

Nearly, if not quite, all the colleges of New England have

adopted the definition of the mathematics requirements formulated, I believe, by the College Entrance Examination Board. We all assume that admission to the study of high school mathematics implies the completion of arithmetic and, naturally, entrance examinations in this subject are no longer set. I sometimes wish they were. These uniform mathematics requirements have been in force so long and have been so freely discussed that one might think that there could no longer be any doubt as to what the colleges expect of the entering student, mathematically. And yet we all know that what they get is still an exceedingly variable quantity,—we might almost say a function of an indefinite number of variables.

In the grades the pupil studies arithmetic for from 6 to 8 years; in the high school he has from $1\frac{1}{2}$ to $2\frac{1}{2}$ years of algebra, 1 to $1\frac{1}{2}$ of geometry and, if he is looking toward a technical course, he adds solid geometry, and trigonometry. Thus he has nearly continuous instruction in mathematics from the beginning of school to the beginning of college. Would it not seem that he should be at least as well equipped in this subject as he is in Latin, which he studies four years, or French, which he has for two or three?

In the *Educational Review*, December, 1911, appears an address given before the American Institute of Instruction by the President of the New England College Entrance Certificate Board. In it are statistics showing the great decrease, under the certificate system of admission, in the percentage of students having an unsatisfactory record during their first semester in college. Figures are given for various subjects for the year 1903-4, and for four years later as follows:

	1903-4.	1907-8.
English	20.2%	7.8%
Latin	14 %	3.7%
Greek	11.8%	2.9%
French	14 %	6.6%
German	13.4%	7.9%
Mathematics	24.6%	13.6%

In all subjects except mathematics and German, the percentage of unsatisfactory records in '07-'08 was much less than half that of '03-'04. Mathematics shows a great improve-

ment, but less than any of the languages, and its percentage of failures is more than double the average for the five language subjects. The last annual report of the Board shows a decidedly improved percentage of failures in mathematics, but still more than double that in the languages, namely, 10.8 per cent. of failures among those admitted by certificate who continue mathematics in college as against 4 per cent. of failures in language. According to the same authority the percentage of failures in mathematics of those admitted by examination was 19 as compared with 9.1 per cent. of those who offered the languages and continued the same language. We still see the percentage of failures in mathematics more than double that in language. Now, these figures are from the product of picked schools, most of which have stood the test of repeated reviews by the board, and the students that have entered college on their certificate have also been carefully selected, only those whose ranks are of A or B grade being, in general, granted a certificate. Except in case of ill health, and such cases would naturally not be reported to count against a school, why should there be any failures to make good, and, in particular, why should mathematics have a larger mortality than language. Almost one ninth of the certified pupils and nearly one fifth of those examined and taking mathematics in college fail to pass. At my own institution the figures are similar, our percentage of failures being somewhat larger. Last year our freshman mathematics for the fall semester was a combined course in algebra and trigonometry and practically one man in four failed to make passing rank. In the college of arts and sciences the record was better, only one in eight falling by the way. This marked difference in the records of our students in our different colleges is probably due to two causes:

1. The crowded curricula in engineering and chemistry, calling for about 28 hours per week of attendance upon college exercises, 13 of which are recitation courses requiring, theoretically, at least, two hours of preparation for one of recitation, making a total working week for the freshman of 54 hours.

2. The fact that many students begin a course in engineering who are not adapted to the work and who, in a college where mathematics is elective, would leave it alone. A considerable

number of these learn their own limitations in this direction and seek a path of less resistance at the end of the first semester.

Who is to blame for this regrettable mortality in freshman mathematics,—the freshman himself, his high school teacher of mathematics, his college teacher, or the system of education? No doubt all of them are responsible, perhaps the high school teacher least of all. The change from high school life with the restraining influence of parents and teachers who have long known them, is a perilous one for many young men, and their inability to use this new freedom discreetly accounts for some of the failures.

Some failures are due, no doubt, to the lack of sympathy between the freshman and his mathematics teacher. The college administration too often feels obliged to put some of these classes in the hands of a graduate student or fellow, or, at best, a teacher of but little experience. The freshman needs, and should have, the benefit of the best teaching talent upon the faculty. At the University of Maine it is our practice to have every member of the mathematics department whose schedule permits it, teach some freshman work. Five of the seven men at the present time giving instruction in freshman mathematics had considerable teaching experience in preparatory schools before taking up college work.

In order to promote closer acquaintance with the students each member of the department holds one hour or more per week open for those who wish to consult him about their work. The students are not only invited, but often required, to attend these conferences. Some of the instructors spend much more than the allotted time with the men. At these conferences an attempt is made to find out what hinders the student's progress,—whether he lacks preparation, is studying too little, or not understandingly, etc. Those needing more help are, of course, advised to tutor.

The constant temptation of the college teacher is to overlook the fact that the freshman's mind is but little more mature and can work but little faster than when he was in the high school. Algebra, trigonometry, elementary analytical geometry, solid geometry, all are so simple to the teacher, why should the student have trouble with them? Perhaps he would not if he really had

the preparation that our stated requirements call for, or assume that he possesses. And this brings me to my subject,—*What Mathematical Knowledge and Ability May Reasonably be Expected of the Student Entering College?*

He has graduated from a school approved by the certificate board or by the department of education of his state, has presented either by certificate or by examination, the full number of units required for admission, including the usual requirements in mathematics; is he ready to enter upon the course in college mathematics with no backward look upon the path he has already traveled?

Let us state some of the things he must know and some of the things he must be able to do if he is to go on successfully. No attempt is here made to follow a logical order or to make the statement complete. First of all, he must have ready for instant use a knowledge of the fundamental facts of arithmetic and must be able to perform its processes with fair rapidity, and be sure that his results are correct. Many students fairly good in written work in algebra are painfully deficient in the solution of easy oral exercises. Personally, I wish that there might be in the high school curriculum a place for the old-fashioned mental arithmetic. I presume that the pages of oral exercises given in our modern text-books of arithmetic and algebra are supposed to take its place, but I fear that many teachers skip these pages. Not long ago a young lady having the reputation of being "the best teacher in town," was conducting a class in arithmetic in the grammar school. The text, an excellent one, was well supplied with oral problems. When asked if she found these exercises helpful she replied: "We do not get time for them." A few minutes later a boy was sent to the board with a problem,— "the area of a circle is 314.16 sq. ft. Find its radius." The pupil divided by .7854, then extracted the square root, then divided by 2. Another boy asked "Why not divide by 3.1416 and extract the root?" The teacher replied: "We had better always follow one rule." Would not she have done better to spend more time on the oral exercises?

Before leaving the study of arithmetic, pupils should be taught the abbreviated processes of multiplication and division and should be required to practice cancellation and a few other

“short cuts.” According to my observation, comparatively few freshmen will divide by four and moving the decimal point instead of multiplying by two hundred fifty, or multiply by 100 and subtract twice the multiplicand instead of multiplying by 98, or choose the easier multiplier when required to find the product of two numbers of, say five figures each, one containing ciphers or repeated digits. Pupils should be required to make rough checks of the reasonableness of their results, particularly in problems involving the placing of the decimal point. If these things have not been done in the grammar school, they should be attended to in the numerical work of geometry and algebra.

In algebra, the aim should be not so much to cover all the ground mentioned in the college catalog requirements, as to master thoroughly the principal subjects, to teach the pupil to reason out all processes and to frequently test results.

Many freshmen have no precise knowledge of the meaning of algebraic terms, but will confuse index and exponent, term and member, and allow the one word simplify to “cover a multitude of algebraic sins.” Some of the most frequent of these sins are: cancellation of a term in the numerator and a term in the denominator of a complex fraction, multiplying one member of a fractional equation by the common denominator of its fractions leaving the other member unchanged, writing $a + b$ for the square root of $a^2 + b^2$.

The pupil who can factor the forms commonly met, change simple surds, whether written with radical signs or fractional exponents, from a given form to another desired form, solve simultaneous equations in one, two or three unknown quantities, form and solve the equations for problems of moderate difficulty, including such as involve radicals and quadratic equations, expand simple expressions by the binomial formula and do these few things rapidly, and with confidence in his results, has the ability to undertake freshman algebra with good prospects of success. The pupil should have learned to read and understand a text-book without the help of a teacher to translate it for him.

In the second course in algebra, or the senior review, it appears to me desirable to teach the use of logarithms in connection with the study of exponents. The pupil will be pleased to learn such a convenient device for finding products, quotients,

powers and roots. I would have this done, not so much for those who are going to college as for the larger number who are not.

In geometry, also, the emphasis should be placed upon quality rather than quantity of preparation. Many of the theorems printed in the average text-book may profitably be omitted. As to the matter of "originals," I should not, perhaps, venture an opinion, as my own preparatory work in geometry was done many years ago with a text-book giving but few of them. But having long felt that the solution of originals was being emphasized far beyond its due importance, I am, naturally, pleased that the "Final Report of the National Committee of Fifteen on Geometry Syllabus" allows me to continue in this opinion. This report which appears in the *MATHEMATICS TEACHER* for December, 1912, and may also be obtained from the United States Commissioner of Education, should be in the hands of every high school teacher of mathematics. Another very helpful pamphlet is "The Report on Entrance Requirements of a Committee of the Society for the Promotion of Engineering Education," which may be obtained, presumably, from the Secretary of that Society, Ithaca, N. Y. The formulation of the mathematical requirements in this report appeals to me as being the most sensible and suggestive that I have seen in print. I would also like to call attention to the Syllabus of Mathematics published by the same society.*

What knowledge and power should the high school boy receive from his study of geometry? He should be familiar with the definitions and properties of the ordinary forms of geometry and should be able to represent them by figures neatly drawn as well as to find their areas, perimeters and volumes with accuracy and certainty. He should know the demonstration of the principal theorems as given in some standard text-book and should be able to give brief and accurate original demonstrations of simple subsidiary theorems.

By omitting the unnecessary theorems and by restricting the number of originals, he will have time for an elementary course in solid geometry. To require solid geometry, is, I suppose, contrary to the movement of the times, but I still feel that the boy who is not going to college should have at least a brief look into

the geometry of space. Leave out the theory of limits, or, at any rate any attempt at proofs, also the incommensurable cases and use the time thus saved to gain knowledge that will really mean something to the boy. After he goes to college he will still find these subjects sufficiently difficult. On the other hand, it is well worth while to teach the high school boy the definition of sine and co-sine, tangent and cotangent and their use in mensuration.

Thus far has been set forth the writer's idea of what mathematical knowledge and ability should be expected of the pupil who is to *continue* mathematics in college. What about the pupil who "does not like" mathematics? Is there a place for him in college, and what mathematics should he be required to do in the preparatory school? The pupils who really cannot learn mathematics are rare; those who, with skillful handling, may not be led to enjoy the subject are comparatively few. All high school pupils should be "exposed" to a year of algebra and a year of plane geometry. If it does not "take," they should not for that reason be refused a high school diploma, nor, in rare cases, admission to college, if they present high scholarship in other lines.

I suppose that every college professor of mathematics has had some painful experiences with students entirely satisfactory in other subjects who have repeated freshman mathematics each succeeding year until graduated "by grace of the faculty." Such cases reconcile us to having mathematics placed among the elective subjects of the curriculum.

In conclusion, let me say that, in my opinion, mathematical teaching in the high school is, at least, as good as that in college; that only a small fraction of freshman failures in mathematics can properly be charged to the high school and that this small fraction may be made still smaller by placing the emphasis of preparation upon quality rather than quantity and by selecting and strongly emphasizing the most vital topics. The boy who knows a few things thoroughly and has really learned to think, and, perhaps, to enjoy thinking in mathematics, is more likely to succeed than he who has merely plodded through the whole text.

UNIVERSITY OF MAINE,
ORONO, ME.